Hands-on: Differential rules for Calculus

Definition

$$f'(x) = \lim \frac{f(x+h) - f(x)}{h}$$

$f'(a)$ is the gradient (slope) of the tangent line to the graph of $y = f(x)$ at $x = a$

Rule	Function	Derivative
Sum/Difference	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
Product	$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
Quotient	$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
Chain	$f(g(x))$	$f'(g(x))g'(x)$
Product of constant and function	$kf(x)$	$kf'(x)$
Constant	k	0
Power	x^n	nx^{n-1}
Generalised power	$(f(x))^n$	$n(f(x))^{n-1} f'(x)$
Exponential (Base e)	$e^{f(x)}$	$f'(x)e^{f(x)}$
Exponential (Base b)	$b^{f(x)}$	$f'(x)b^{f(x)} \ln b$
Natural Logarithm	$\ln f(x)$	$\frac{f'(x)}{f(x)}$
Logarithm	$\log_b f(x)$	$\frac{f'(x)}{f(x) \ln b}$

Differential

$dy = f'(x)\,dx$ Relative error in y is $\frac{dy}{y}$

Newton's method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, $n = 1, 2, 3, \ldots$

Hyperbolic functions

$\sinh x = \frac{ex - e^{-x}}{2}$	$\cosh x = \frac{ex + e^{-x}}{2}$	$\tanh x = \frac{\sinh x}{\cosh x}$
$\operatorname{cosech} x = \frac{1}{\sinh x}$	$\operatorname{sech} x = \frac{1}{\cosh x}$	$\coth x = \frac{1}{\tanh x}$

TRIGONOMETRIC FUNCTIONS		HYPERBOLIC FUNCTIONS	
Function	**Derivative**	**Function**	**Derivative**
$\sin x$	$\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$-\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$\sec^2 x$	$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$	$\operatorname{cosec} h$	$-\operatorname{cosech} x \coth x$
$\sec x$	$\sec x \tan x$	$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\cot x$	$-\operatorname{cosec}^2 x$	$\coth x$	$-\operatorname{cosech}^2 x$

INVERSE TRIGONOMETRIC FUNCTIONS		INVERSE HYPERBOLIC FUNCTIONS	
Function	**Derivative**	**Function**	**Derivative**
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\operatorname{cosec}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\operatorname{cosec} h^{-1}$	$-\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$
$\sec^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\operatorname{sech}^{-1} x$	$-\frac{1}{x\sqrt{1-x^2}}$
$\cot^{-1} x$	$-\frac{1}{1+x^2}$	$\coth^{-1} x$	$\frac{1}{1-x^2}$

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